Elliptical Arch Howe Truss Popsicle Stick Bridge

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Contents:

Goal:

Design a popsicle stick bridge to be tested destructively to demonstrate learning that has to support a large weight through the middle, using only 100 unmodified sticks and white glue.

General Research:

First, we consider what we intend to do before searching: Make a popsicle stick bridge. So given the nature of the medium, and that we will be testing to failure, we should determine a methodology that best utilizes the materials.

Results:

Many sources (Hammer, 2008) suggest that popsicle sticks are stronger when bent along their edge. This makes intuitive sense, the grain of the material is being stretched along its length, vs perpendicular to it. Wood fibers are very strong when stretched, so when designing, we should always attempt to make our connections in a way that keeps the forces acting upon the sticks in a favourable fashion.

Choosing Bridge Type:

Knowing this, we can investigate what parts of a bridge are stressed the most in certain types of bridges to pick one that has the highest forces in the members, on those in tension. Using a software called Bridge Designer 2016 we tested various bridge designs, changing only the geometry of the members, using a member length that matched the scale of our popsicle sticks. From this, we determined the Howe truss to be the best, having the best performance as weight increased, due to a majority of the higher magnitude forces being in tension, rather than compression.

Applying this:

Now we can proceed with concept generation, as we know the general geometry that we should follow when designing the bridge. However, there are many things left to figure out, such as the methodology of joints, or how the chords should be constructed.

Concepts:

Generating concepts:

With the geometry of the Howe truss in mind, here is what we came up with for the first round of testing. Initially there were some considerations of no howe trusses, but they were quickly invalidated:

- Double offset chords for maximum glue surface area and strong joints:

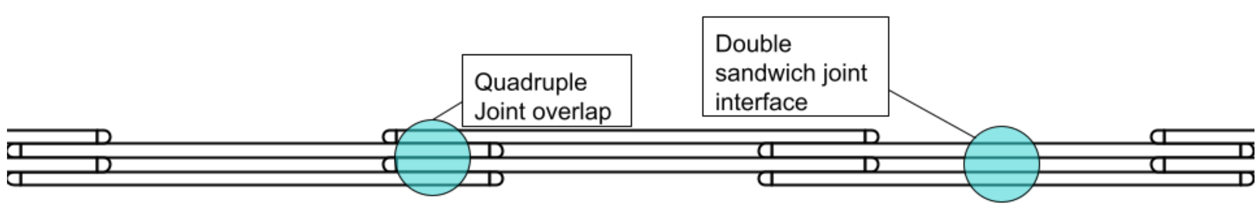


Fig.1.1: Top view of proposed chord design with large surface areas for connections

- Bent chords for contained joints:

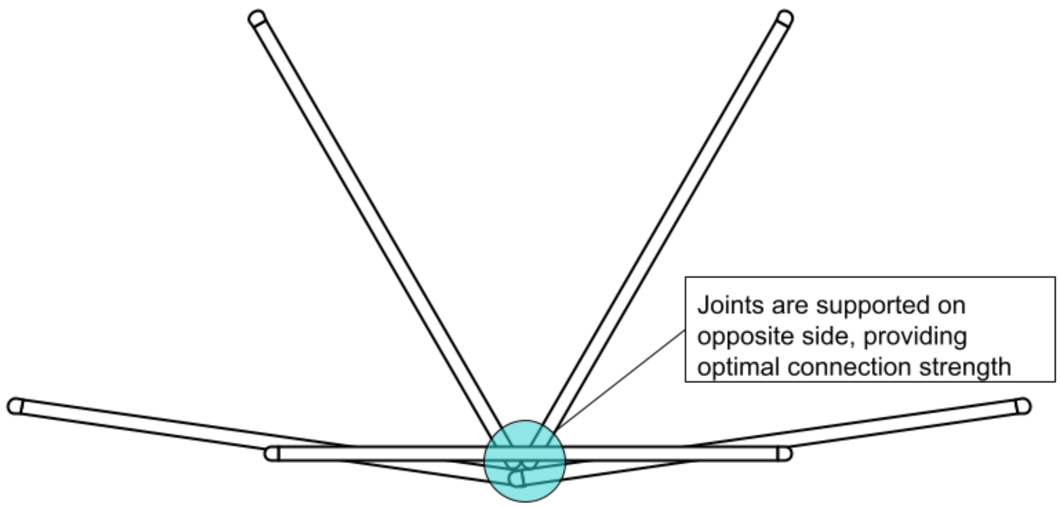


Fig.1.2: Side view of possible joint and chord design that contains the opposite end of the member.

- Triangular Bridge for stick efficiency and torsion prevention (parallel sway movement):

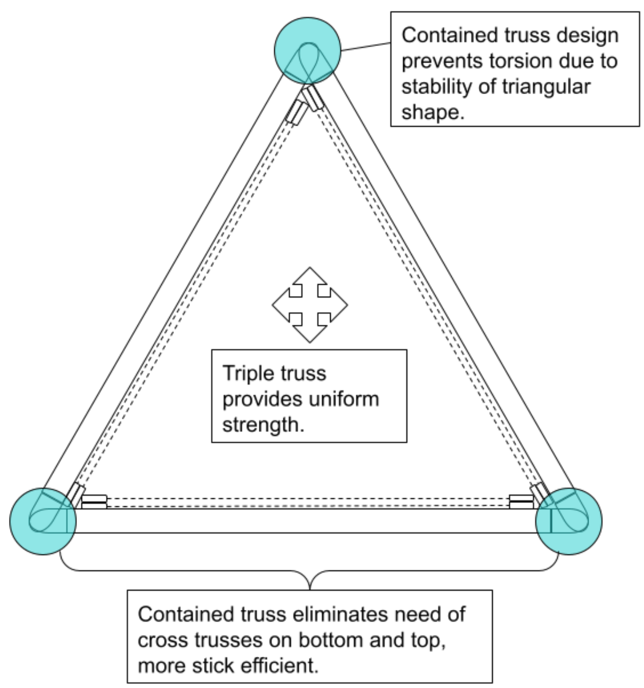


Fig.1.3: Triangular Bridge concept, intended to reduce stick usage and provide a stronger, more stable shape of the overall structure that is not as sensitive to non linear loading.

Consideration, Testing, and Initial Selection of Concepts:

The double offset chords concept quickly became a cornerstone of the project, for which all subsequent concepts relied upon. We compared the strength to a single width chord, as well as the ease of which the members could be removed from it. By simply encasing the other face, it became difficult to remove the member from within without damaging the sticks before the glue connection failed, and in some cases, impossible. From this, we made an early decision to use this concept for all subsequent concepts and iterations.

The contained joint concept had its merit in its extra dimension of joint security, but due to the angles required, when extended into multiple sections, would lead to a highly asymmetrical and inefficient design. However as we will see later, the arch concept will be revisited.

The Triangular bridge after minor testing turned out to be a complete failure, and with hindsight, it should have been obvious of this failure. Due to the force loading vertically, the sides of the triangle will naturally tend to be pushed away and out to the side like this:

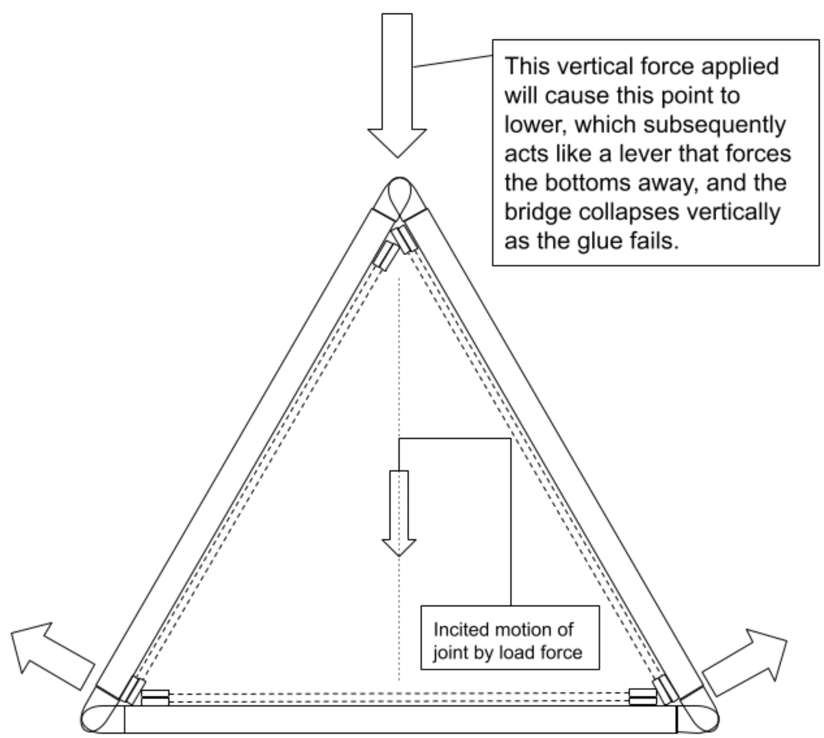


Fig. 2.0: Forces in the triangle bridge that caused its failure.

Seeing that it failed completely, we moved onto the design of a simple Howe truss bridge, utilizing the double sandwiched chords concept.

Design Process:

Design Variable Determination:

Stick Overlap

The amount of overlap needed is determined by how much glue surface area we need at a joint for the member to fail before the glue. Testing to failure, 35mm overlap had the highest length to weight supported ratio. (Hammer, 2008)

Overall Length:

The overall length of the bridge is defined by how many sticks are used along the bottom chord, and by the overlap between them. The formula for the total length T using n sticks that are of length L with an overlap of o is: T= (L* n) - ((n - 1) * o)

$$T = (L \times n) - ((n - 1) \times o)$$

This must be greater than 500mm, and we know that o = 35mm. Subbing in for L = 113.38mm:

$$500mm < (113.38mm \times n) - ((n - 1) \times 35mm)$$
$$500mm < 113.38n - (35n - 35)$$
$$500mm < 78.38n - 35\text{mm}$$
$$535mm < 78.38n$$
$$\frac{500mm}{78.38} < n$$
$$n > 5.93mm$$

Since we know n must be an integer, we round to $n = 6$. Lets see by how much we clear the gap of 500mm, the total overhang o_h:

$$o_h = T - 500mm$$
$$o_h = (113.38 \times 6) - ((6 - 1) \times 35mm) - 500mm$$
$$o_h = 505.28mm - 500mm = 5.28mm$$

Determining the Number of Bottom and Top Sticks:

An overlap this small would leave less than half a cm on the table on either side, and as the bridge flexes, it shortens. We need to use one more stick to provide a proper contact with the table on both sides. Using 7 sticks on the bottom gives us 583.66mm in total, or 41.83mm overhang on both sides, which is acceptable. To conserve on sticks, we will use 5 sticks of length on the top, centered on the 7 below.

Digital Prototyping I:

With the design variables determined, we created a digital prototype to test on, and do simulations with. The following model was modeled and simulated in Autodesk Fusion 360:

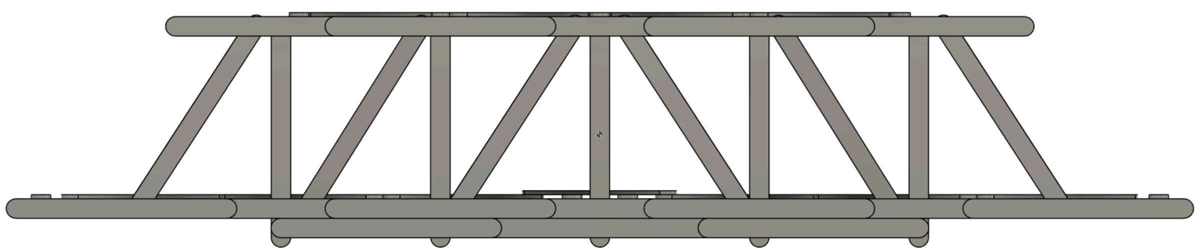


Fig.3.0: Side view of first design. Note the placement of extra sticks on the bottom under tension.

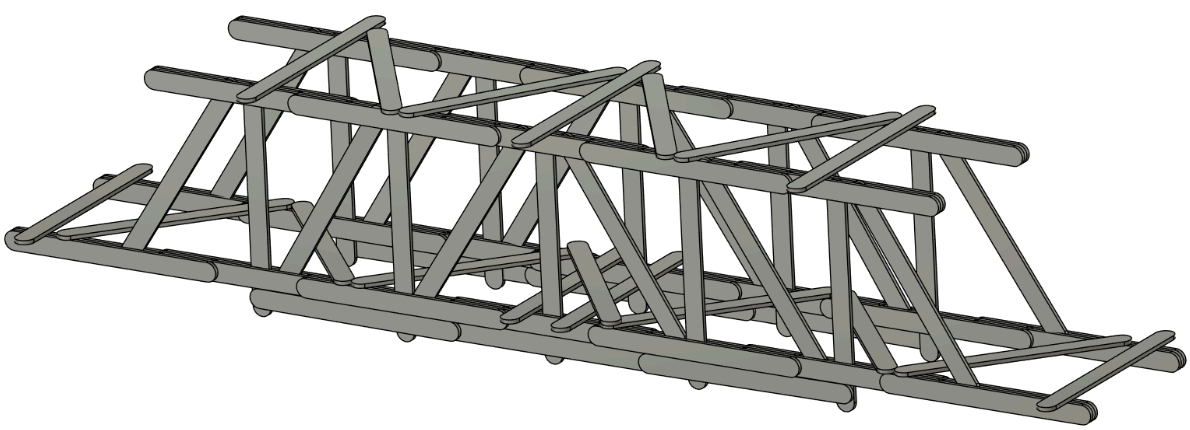


Fig.3.1: Perspective view of first design.

In making these prototypes digitally, the specific locations and offsets of the members were determined through geometric constraints within the software, and using the model as a blueprint, the first physical prototype was ready to be constructed.

Prototype:

The first prototype was mostly a breeze to build, but there was a lot learned from its construction that will guide the final revision.

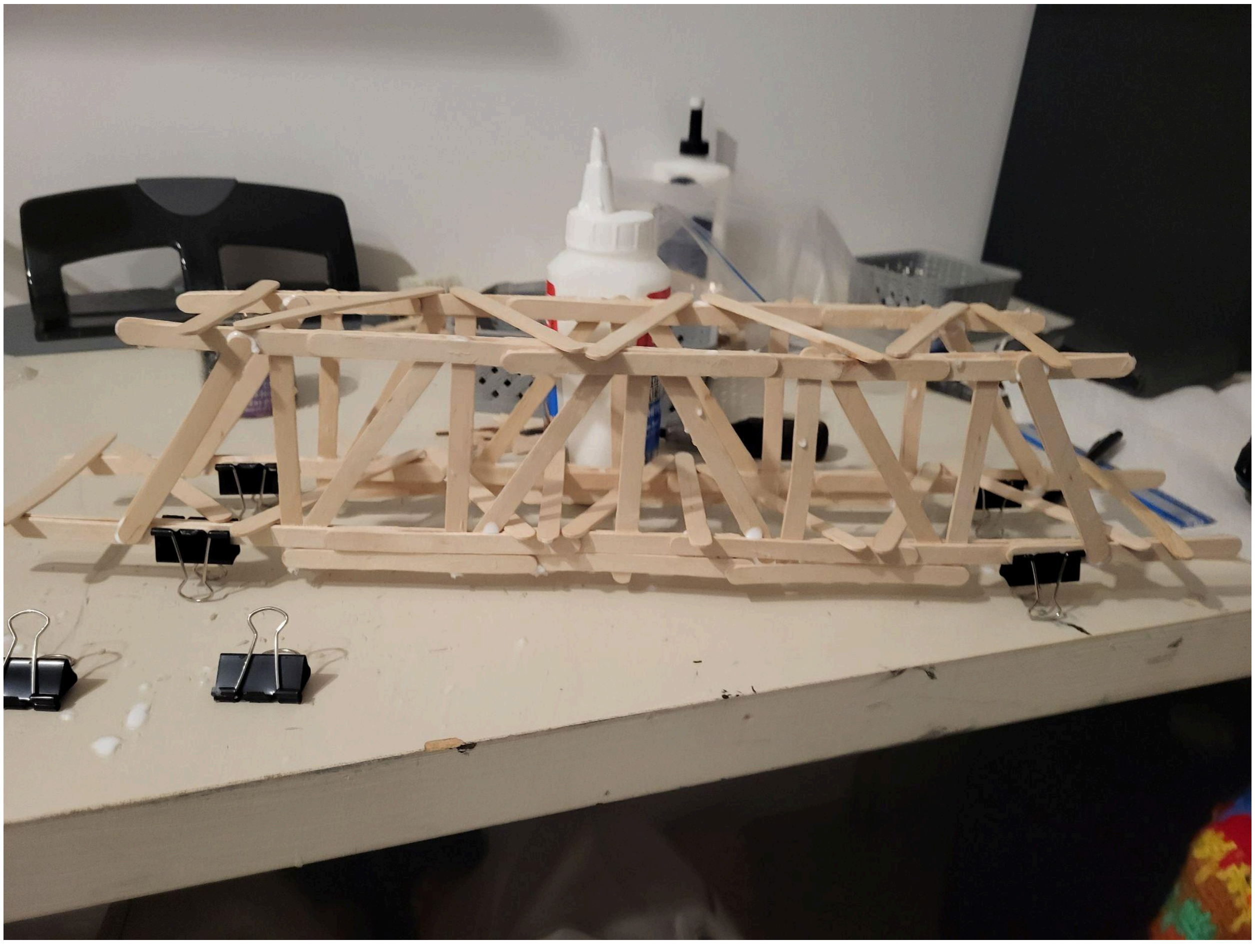


Fig.4.0:The finished prototype out to dry

Construction Methodology and Challenges:

When building this prototype, there were some great annoyances and things that would, in the end, prove detrimental to the quality of the final model.

- Keeping the lower and upper chords parallel when starting the process of joining with members:

This was exceptionally frustrating, but after some experimentation, we found that by tacking the top and bottom in place with one side fully clamped, it allowed us to make the vertical member perpendicular with a square by slightly reducing pressure on the clamp and nudging it into place.

Then this was repeated for the other edge for full perpendicularity on the middle member, which in turn made the top and bottom chords parallel.

- Inserting the members while retaining proper glue distribution within the joint:

  When inserting members into the sandwich joint, it was difficult to ensure both faces of the stick interfaced properly with the chord joint space, as when it was pushed in, it tended to push all of the glue out. We solved this by using a toothpick to distribute the glue fully between the joints, then insert the members diagonally along the length to minimize the dragging effect of inserting the member.

- Keeping the two trusses aligned in all directions:

  When attempting to connect the two built trusses together, it was a massive struggle to maintain alignment. At this stage, this wasn't solved, and resulted in a slightly bent structure, leading to structural problems when testing.

Information Learned From the First Prototype:

What can be Fixed/Improved? How?

The most immediate problem was the alignment between the two trusses. Due to issues during construction that were not immediately obvious how to solve, the two trusses were visibly skewed when looking down along its opening. The issue is demonstrated as so:

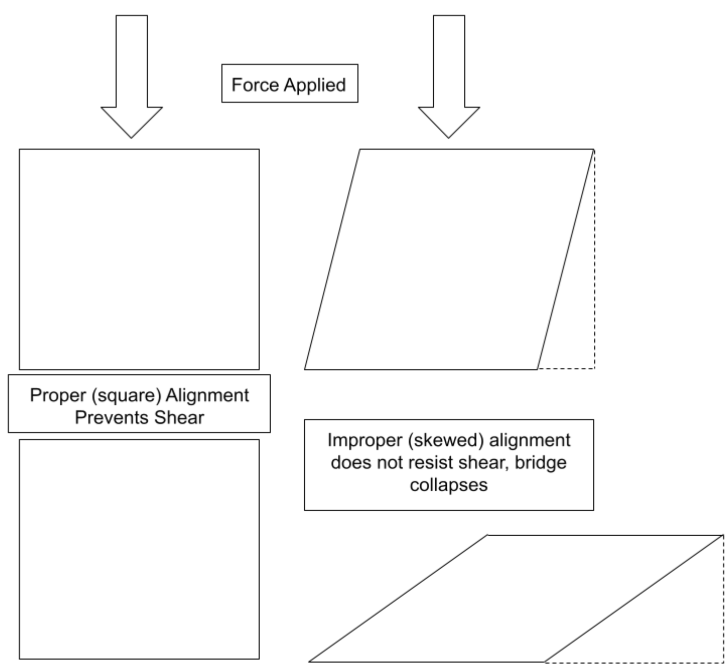


Fig.5.0: Exaggeration of the misalignment.

Along with this, it was seen that as the model bent under load, it progressively got weaker. To combat this, and have the peak loading of the model be when it is flat, we can give it some curvature to take the preload, and have the higher load be when it is flat, at a later weight than if it were initially flat.

With all of these considerations in mind, it is time to work on the final iteration.

Design Variables of Final Iteration:

For the final iteration, we want to implement an arch to be able to have a preload on the structure. With this comes the following values that must be determined:

1. Arch Type
2. Arch Deflection
3. Arch Radius (we find this to be the same as deflection later)

To determine these, we can utilize Python and a plotting library to plot the radii and deflections for a given force:

```
import matplotlib.pyplot as plt

# Function to calculate the force required to flatten the chord and the
radius of curvature
def calculate_flattening_force_and_radius(span_length, deflection, E, b,
h):
    """
    Calculate the force required to flatten a curved chord and its radius
of curvature.

    Parameters:
    span_length (float): Length of the span (L) in mm.
    deflection (float): Initial deflection (δ) in mm.
    E (float): Modulus of elasticity (N/mm²).
    b (float): Width of the popsicle stick in mm (horizontal dimension).
    h (float): Height of the popsicle stick in mm (vertical dimension).

    Returns:
    tuple: Force required to flatten the chord (N), Radius of curvature (R)
in mm.
    """
    # Calculate the radius of curvature (R) based on the span length and
```

```
deflection
    R = (span_length**2) / (8 * deflection) + (deflection / 2)

    # Calculate the moment of inertia (I) for the revised orientation
    I = (b * h**3) / 12  # mm⁴

    # Calculate the bending moment (M)
    M = E * I * (1 / R)  # N·mm

    # Calculate the force required to flatten the chord
    force = (4 * M) / span_length  # N

    return force, R

# Define constant parameters based on updated values
E = 5000  # Modulus of elasticity (N/mm²) - typical for wooden sticks
#(Sautel et al., 2017)
b = 2     # Width of popsicle stick in mm (horizontal dimension)
h = 9.58  # Height of popsicle stick in mm (vertical dimension)
span_length = 500  # Span length of the bridge in mm

# Define a range of deflections to test (from 5 mm to 100 mm in steps of 5
mm)
deflections = list(range(5, 105, 5))

# Calculate the force and radius for each deflection
forces = []
radii = []
for delta in deflections:
    force, radius = calculate_flattening_force_and_radius(span_length,
delta, E, b, h)
    forces.append(force)
    radii.append(radius)

# Print the results
print("Force and Radius of Curvature for Different Initial Deflections
(Using Corrected I)")
print("--------------------------------------------------------")
for delta, force, radius in zip(deflections, forces, radii):
    print(f"Deflection: {delta} mm -> Force: {force:.2f} N, Radius:
{radius:.2f} mm")

# Plot the force vs deflection graph
```

```
plt.figure(figsize=(12, 6))
# Plot force vs deflection
plt.subplot(1, 2, 1)  # Create a subplot for force vs deflection
plt.plot(deflections, forces, marker='o', linestyle='-', color='g',
label='Force vs Deflection')
plt.title('Force Required to Flatten the Curved Chord')
plt.xlabel('Initial Deflection (mm)')
plt.ylabel('Force Required (N)')
plt.grid(True)
plt.legend()
# Plot radius vs deflection
plt.subplot(1, 2, 2)  # Create a subplot for radius vs deflection
plt.plot(deflections, radii, marker='o', linestyle='-', color='b',
label='Radius vs Deflection')
plt.title('Radius of Curvature for Different Deflections')
plt.xlabel('Initial Deflection (mm)')
plt.ylabel('Radius of Curvature (mm)')
plt.grid(True)
plt.legend()
# Show the plot with both subplots
plt.tight_layout()
plt.show()
```

This gives us these graphs:

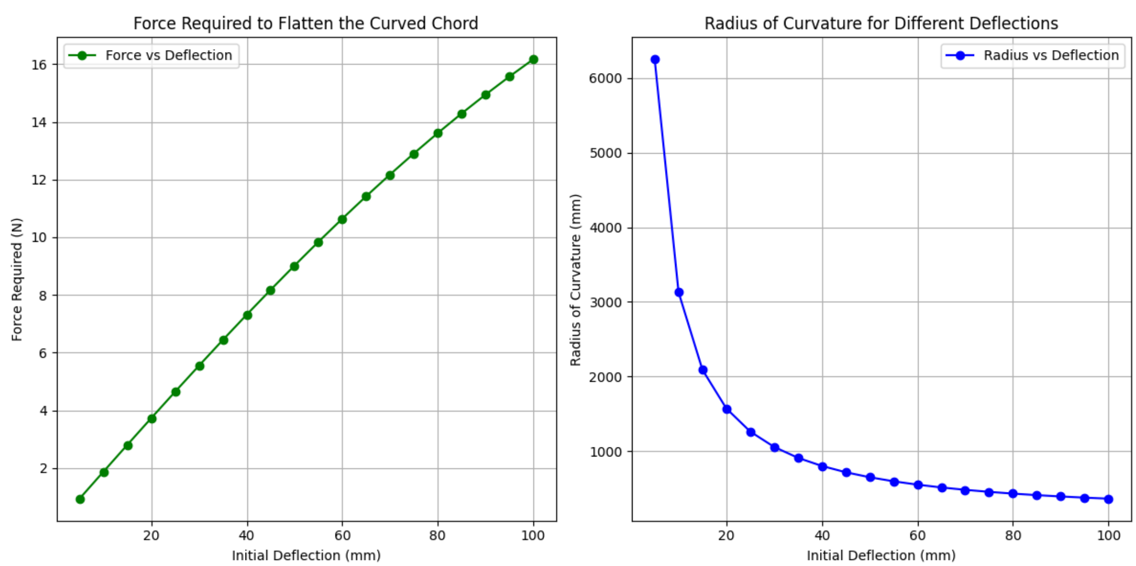


Fig.6.0: Force Required Flatten vs Deflection and Deflection vs. Radius of Curvature

Determining Arch Type:

Before analyzing these trends, we can find the arch type that will fit our needs. We can do this by noting that our span distance is 500mm: we rule out circular arches based on the fact that the bridge would then need to have a deflection of 250mm, and would cause members to alter their effectiveness by making the outer edges always in compression. So considering this, we choose an elliptical shape, with its major axis as 500mm and its minor as the deflection value. The tapering shape will make it a gradual change in steepness and will allow proper preload without causing detriment to the effectiveness.

Determining Deflection:

Analyzing the trends, we can see some linearity in the rate of Radius vs. Deflection between 100mm and 40mm. This makes sense because as the initial deflection gets small, the radius will grow to infinity because the initial deflection approaches a flat line. However, we can't choose too large of a deflection such that the ends are at too steep of an angle relative to the center, or such that the force to flatten it is too large. We must also consider that this figure is underguessing the force required since it is only modeling an ideal, single chord with an ideal central load. In reality, we will have multiple chords on trusses that are wider. With this in mind, 50mm of deflection was chosen to keep the bridge at a non extreme angle at the end, while still giving ample preload benefits. Because we have an elliptical arch, the radius really is simply a metric of how steep the ends of the model get instead of an actual circular radius.

Concepts Specific to Final Iteration:

Perpendicular Joining Members for Shear Prevention:

Earlier we faced issues keeping the chords parallel. In this iteration, we fix this by adding perpendicular members across the top in addition to the standard flat members. These members are in a far more optimal orientation, providing exceptional shear resistance.

Extended outer rails for better force distribution and leveling:

Due to the model having an arched shape, the ends of the model would only rest on a small tangential region on the ends. To increase the surface area in contact with the table for the initial force before it inevitably bends upward (could be prevented by putting a bend prevention member that would touch the side of the table, but by convention this is not allowed in nearly all popsicle stick assignments/competitions). The extended arms also allow for a corrective measure of leveling in the end, after the minor errors of construction have added up and will allow a nicely level final product.

Digital Prototyping II

With the design specific variables and concepts defined, a digital prototype can be constructed to validate geometry and be utilized as an aid in construction. The following model was modeled and simulated in Autodesk Fusion 360:

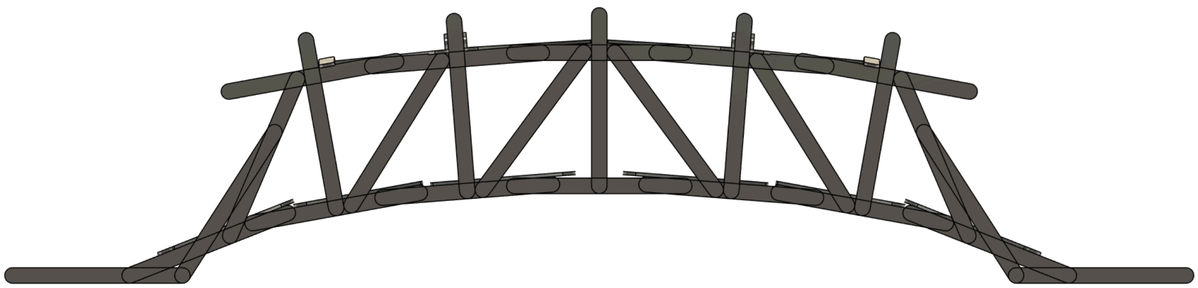


Fig.7.0: Side view of elliptical Howe truss bridge. Note the leveling extensions on the ends.

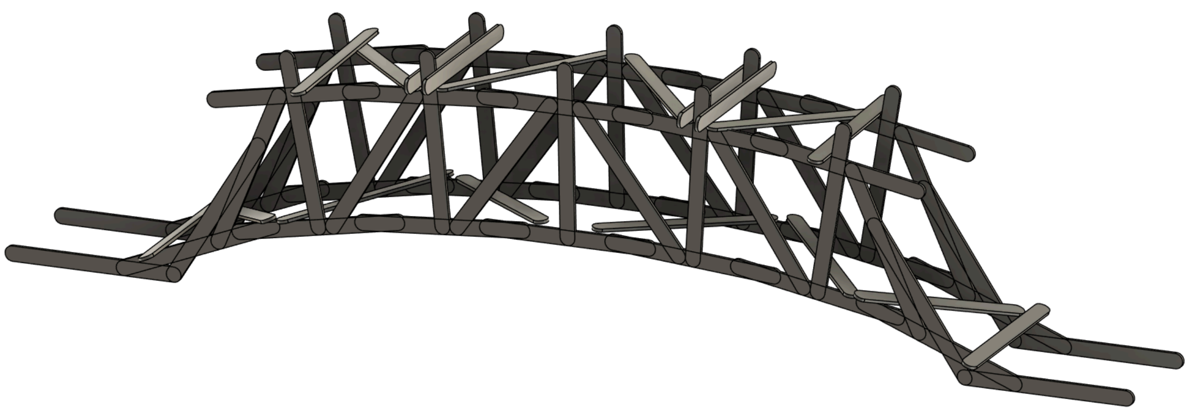


Fig.7.1: Perspective view of elliptical Howe truss bridge. Note the perpendicular cross members along the top that reinforce the model from shearing, and the central void for the weighing apparatus.

Better Team Communication:

Due to the more complex nature, more robust construction documentation was collected, and a dynamic drawing of the geometry was created for referencing during construction.

Construction of Final Iteration:

This is a far more complex construction than before. Various strategies were employed to maintain alignment in all dimensions.

- Horizontal Leveling:

The trusses were kept horizontally level, that is, a constant width between them, by using a variable width shim fashioned from two bull clips, and measured to the virtual prototype's width using calipers and adjusting the inner bull clip:

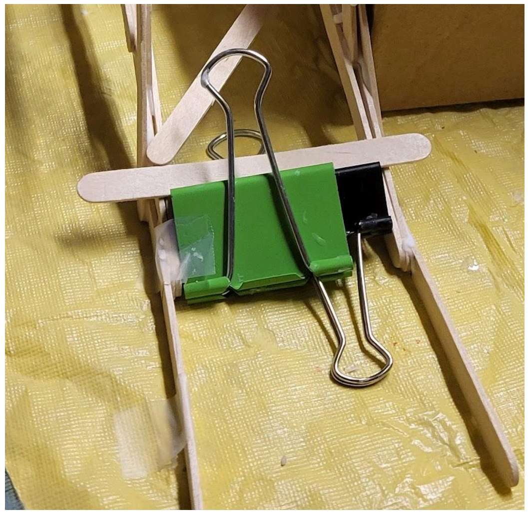


Fig.8.0: Horizontal shim used to maintain constant width, tacked in place with tape.

- Vertical leveling:

The trusses were leveled vertically through accurate construction, and by minutely adjusting the relative heights of the end members pictured in Fig.8.0. But when it came to joining them together, some assistance was necessary:

Fig.8.1: A box is utilized to keep the back edge at a reference vertical.

To maintain verticality in the other truss, we tacked the perpendicular top members in with bull clips as so to get the whole model square:

Fig.8.2: Perpendicular members holding the bridge square.

- Longitudinal leveling:

The trusses were leveled longitudinally by simply using a square edge at the back end where the end members rested. In this case, a ruler was used to level them, and then they were taped in place:

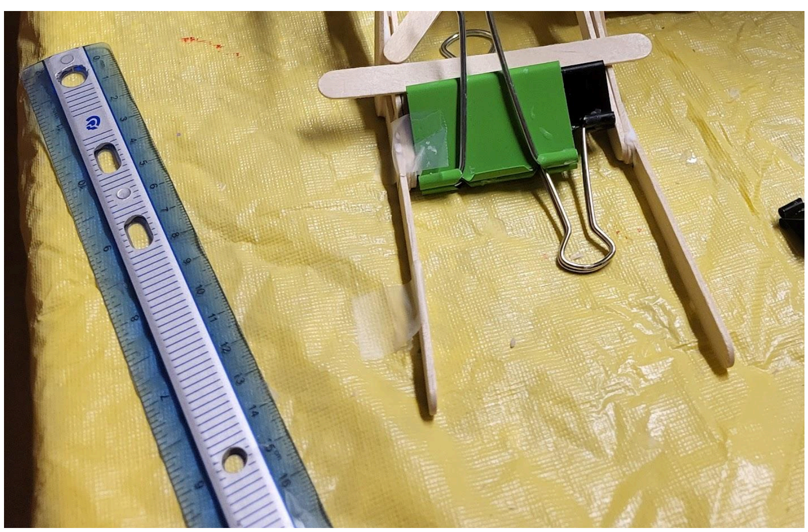


Fig.8.3: Ruler is used to align back edges to the same length.

Finishing Touches:

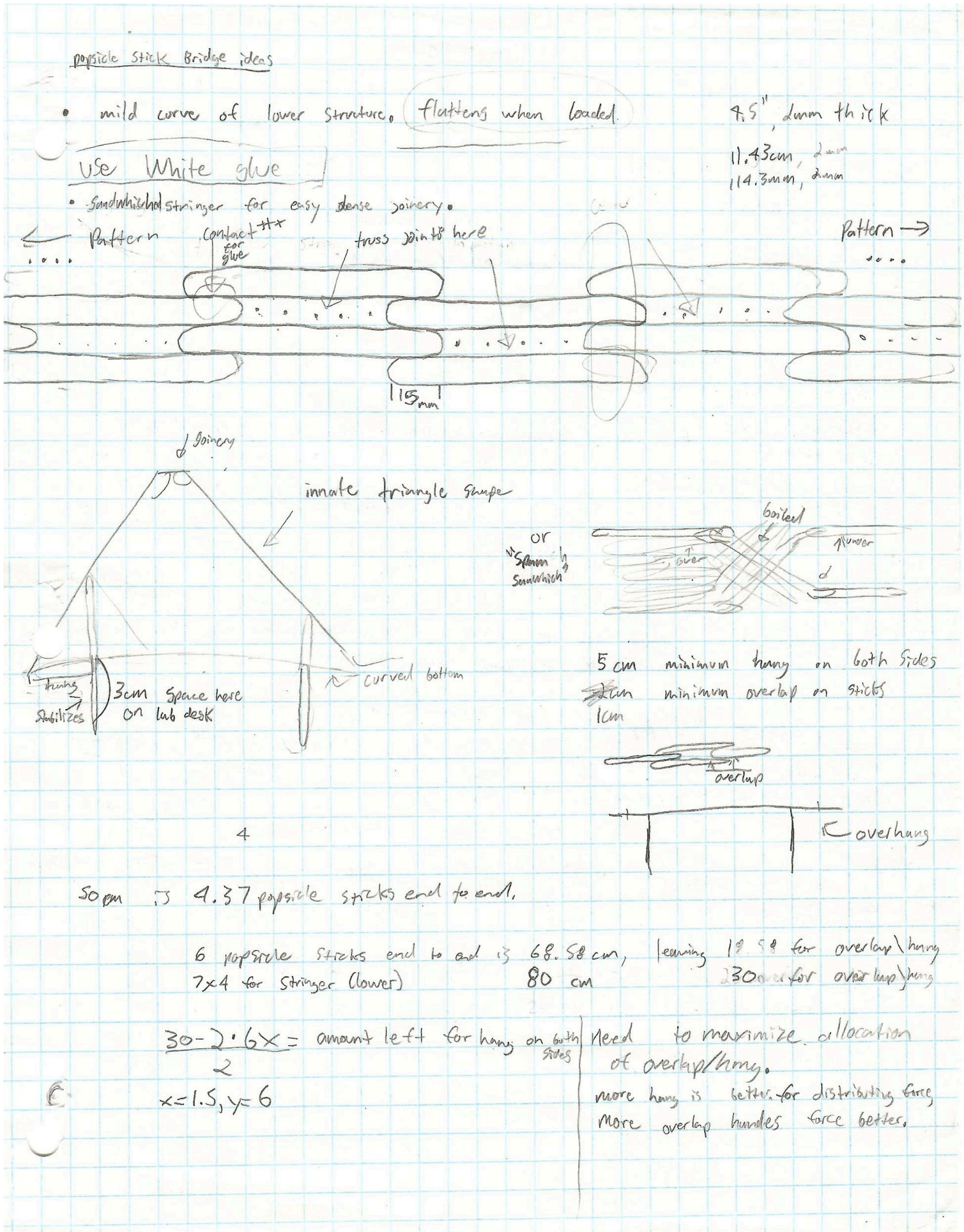
After completing the structure, a road was added for finishing:

Fig.9.0: Newly completed bridge with its road.

And later, it was painted:

Fig.9.1: Painted Bridge.

Design Sketches:



inputs: (constant)

popsicle stick width : 9.58mm
length : 113.38mm
thickness : 2mm

Span: 500mm

Parameters

1) # bottom layer sticks
2) top
3) overlap

Results

4) hang
5) overall length
6) Joint space

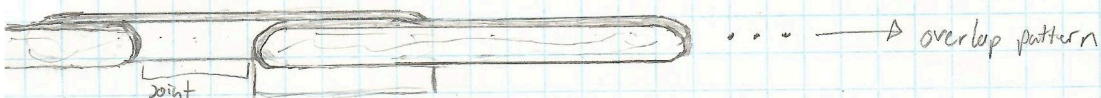
Definitions:

1, 2) # bottom / top

The number of sticks long a respective layer is, not how many a layer contains.

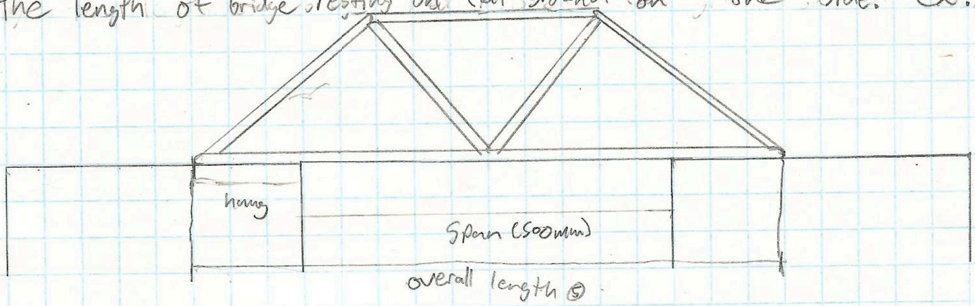
3)

The overlap length for glueing. ex:



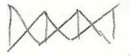
4) hang

The length of bridge resting on flat ground on one side. ex:



possible

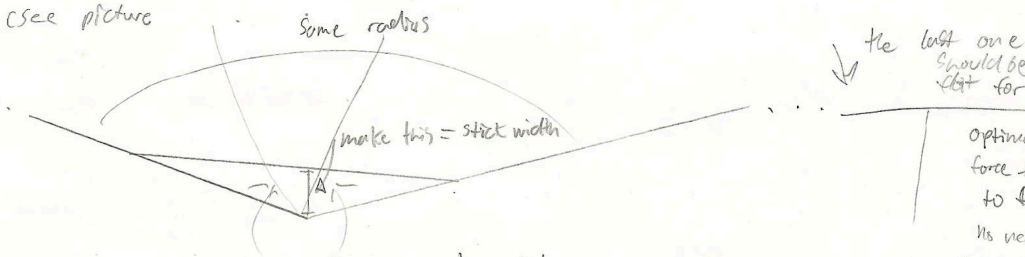
Focus on thickening Rails.

 - - - truss pattern with center vertical beam

or use overlap arch for pin joint connections that are supported below.

(see picture

Same radius



make this = stick width

the last one should be flat for optimal force transfer to table, no need to support member below or for double pin connection since

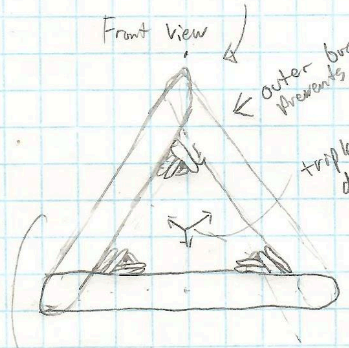
members meet at a point, and are supported below.

The ☐ design had Shear issues.

likely use howe truss to put highest stress members in tension, many sources find sticks to be stronger in tension than compression.

lets try to use Δ to fix Shear problem to give the truss more of the force.

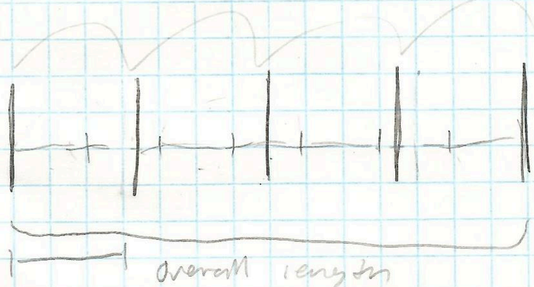
Closed edge top geometry wastes less space, more sticks in more optimal orientations



use sandwhich truss design for chords.

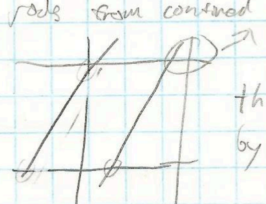
rib Shell unifies Shaping to keep forces contained along the member

Ribbing count = # sticks long -2



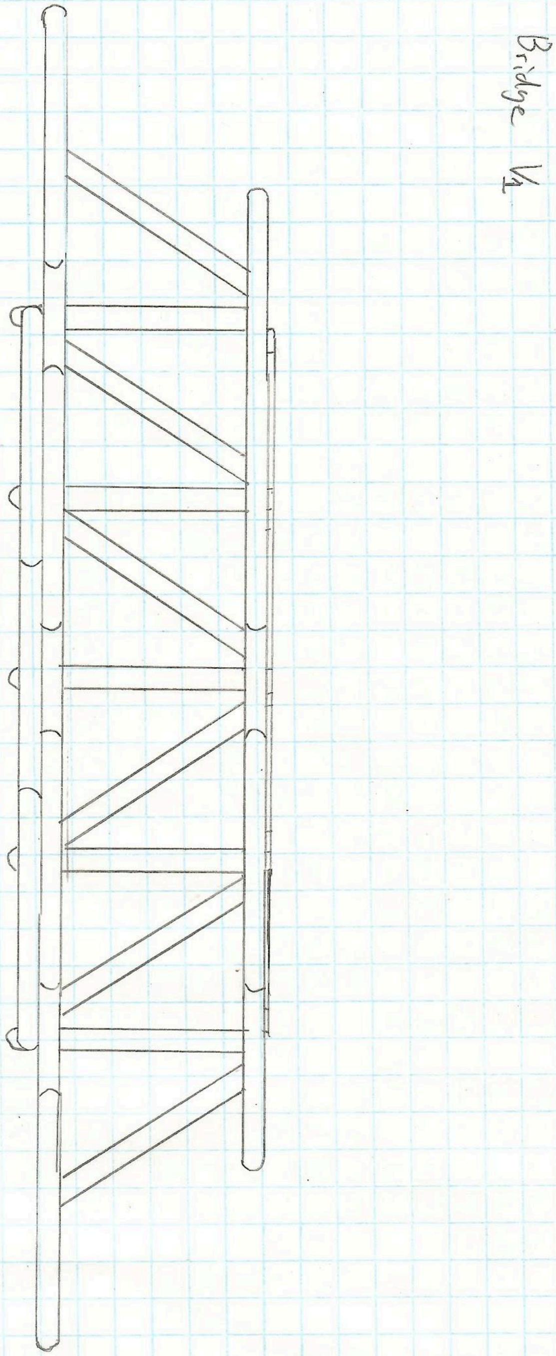
Spacing = length / (Ribbing count - 1)

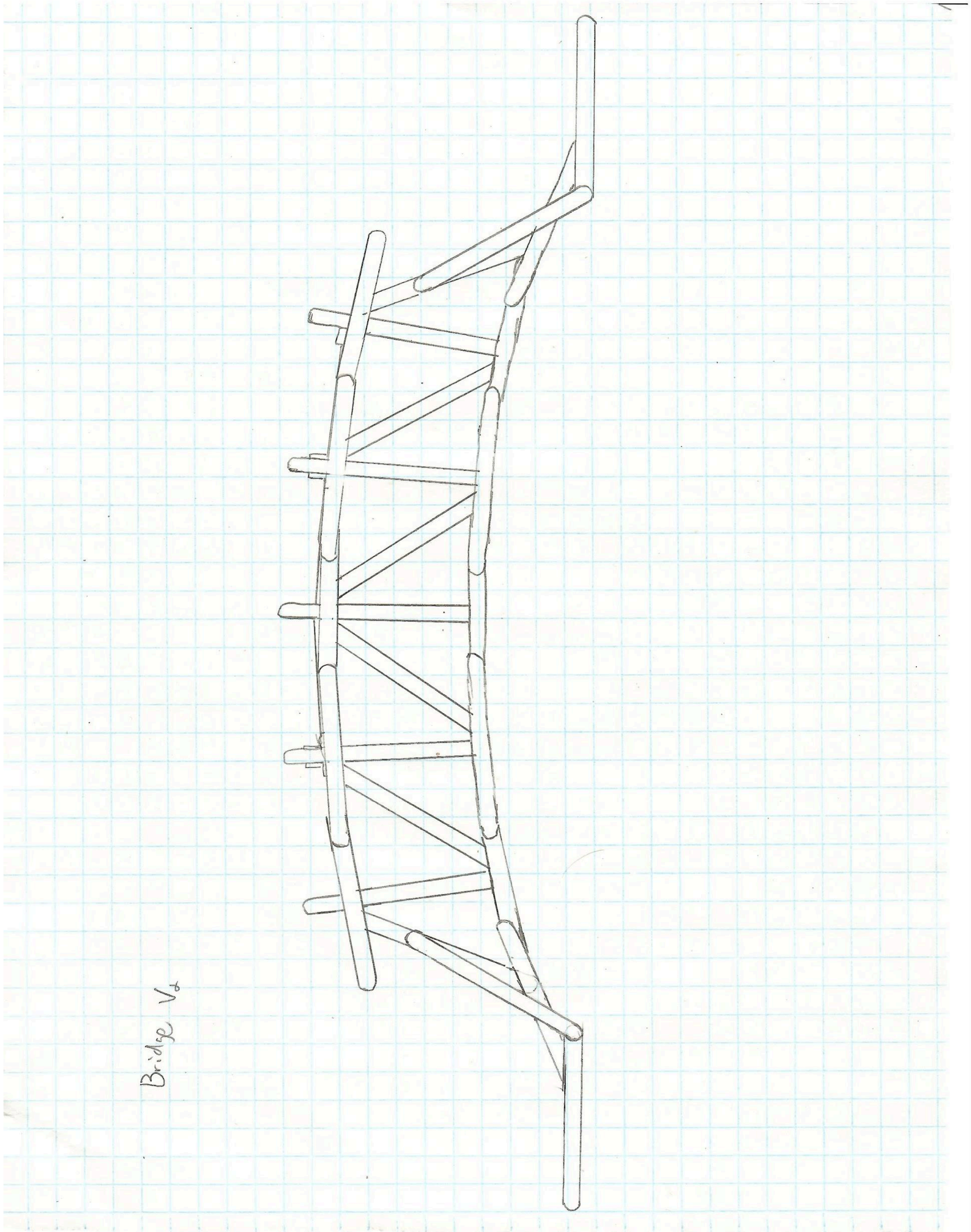
* may have to change Spacing to accomodate protruding struts/ rods from contained length



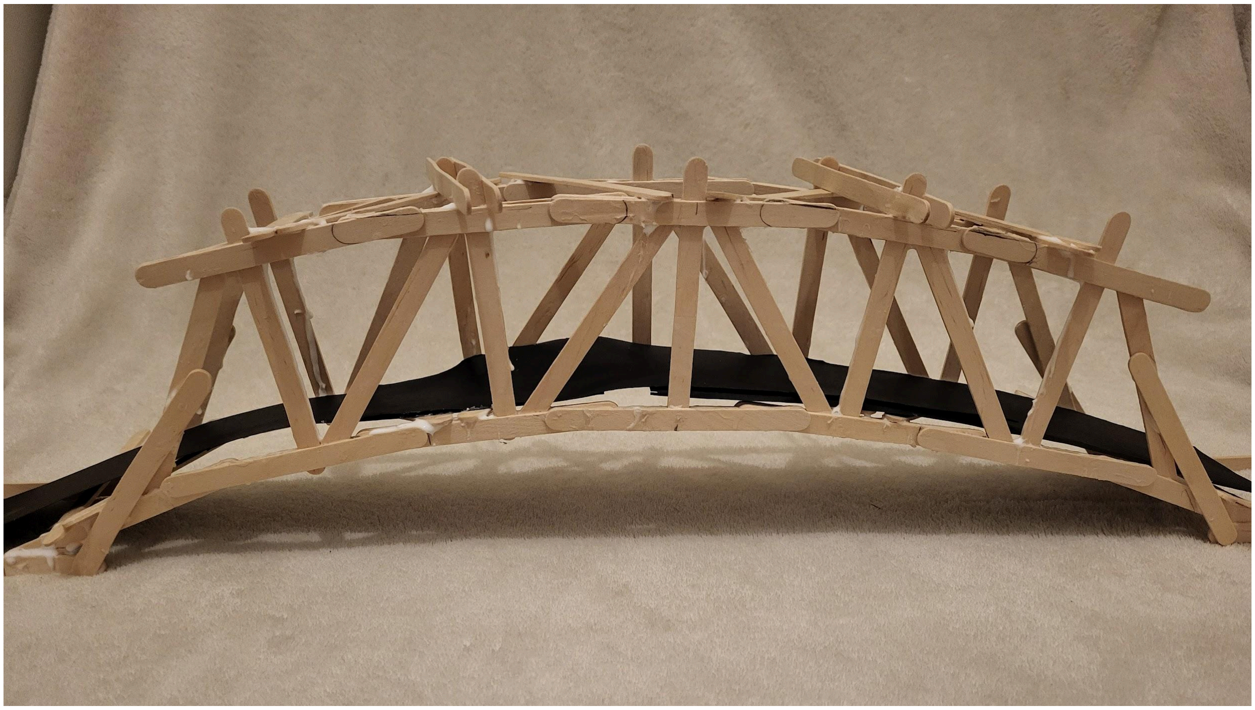
this is caused by the Δ Shell,

the ch


Bridge V1


Bridge V2

Final Product:



References

Hammer, J. (2008, April 7). Small Scale Bridge Design. Ottowa, Ontario; Carleton University.

Sautel, J., Bourges, A., Caussarieu, A., Plihon, N., & Taberlet, N. (2017). The physics of a popsicle stick bomb. *American Journal of Physics*, *85*(10), 783-790. https://doi.org/10.1119/1.5000797